Activity 1: Down the Drain: Linear Function – Negative Slope

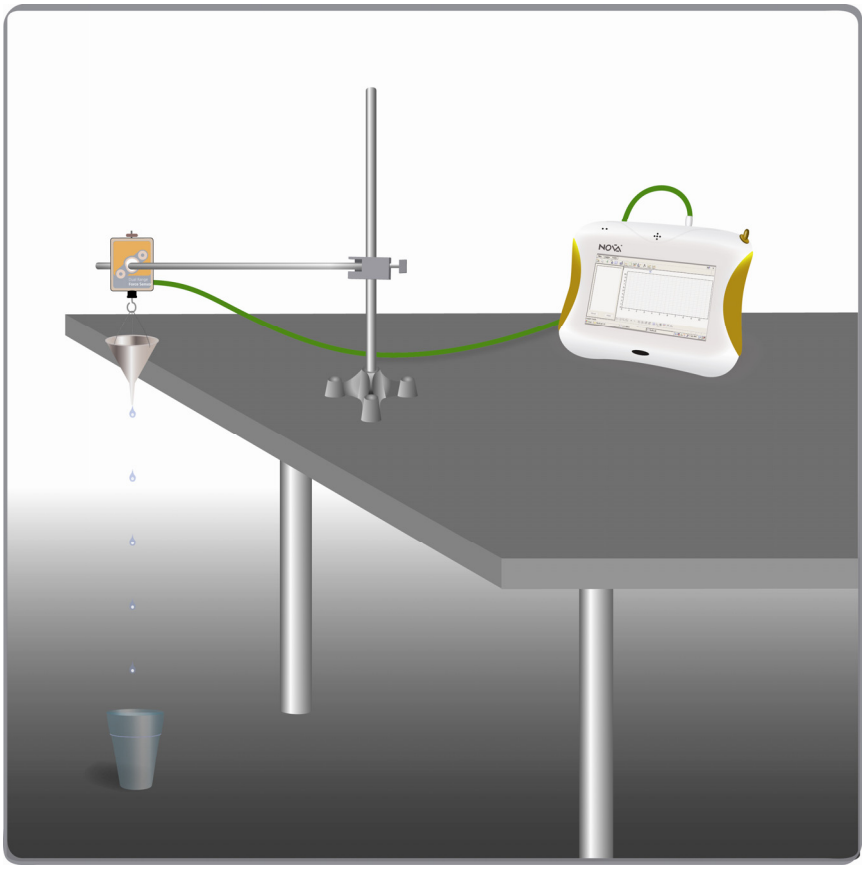

Figure 1-1. Down the drain

Introduction

Consider a container filled with water. The water drains through a hole in the container. What parameters control the process?

In this activity you will use the Force sensor to fit a linear function which has a negative slope to the decreasing water weight vs. time graph. You will then analyze the graph in order to write the slope/intercept form for the given graph.

The equation $f(x) = mx + b$ represents a linear function. Linear functions are functions that have *x* as the input variable, and *x* is raised only to the first power. The linear function's graph is a straight line (hence the term "linear").

A **linear function** can be written in the following forms:

f(x) = mx + b	Function form	Example: f(x) = 3x - 1	m = 3, b = -1
y = mx + b	Equation form	Example: y = 3x - 1	

where **m** and **b** are fixed numbers (the names m and b are traditional).

The equation form is known as the slope/intercept form of linear functions. In the x-y plane linear functions are represented by straight lines. Here *m* is the slope of the line and *b* is the point where the line crosses the y-axis (known as the y-intercept). The x, y coordinates for the y-intercept are (0, ***b***).

Slope measures the rate of change in the dependent variable as the independent variable changes. The greater the slope, the steeper the line.

Slope shows both steepness and direction. With negative slope the line moves down when going from left to right, the function decreases when moving from left to right.

If (x_1, y_1) and (x_2, y_2) are two points on the line, then the slope is:

$$(1)\ m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

The intercept is the y coordinate of the point at which the line crosses the y-axis, $(0, b)$. In this activity the slope of the linear fit is negative.

Learning Objectives

1. To write a linear equation in slope/intercept form using two different methods:
   - Manually calculating the slope and y-intercept
   - Plotting the manual line fit on the graph
   - Applying automatic curve fit
2. To evaluate the two methods to fit the linear function

Equipment

- Nova5000
- Force sensor (±10N)
- Plastic funnel
- String
- Table stand, right angle clamp and rod
- Water
- Bucket or Container

Equipment Setup Procedure

1. Use a sharp object, such as like a nail, to make three small holes in the funnel. The holes must be equally spaced, next to the top rim of the funnel as shown on Figure 1-1.
2. Tie a piece of string to each hole and then tie the three strings together.
3. Assemble the equipment as shown on Figure 1-1.
4. Place a bucket or other container on the floor beneath the funnel to contain the falling water.
5. Start **Multilab**.
6. Plug the Force sensor into input 1 (I/O-1) of the Nova5000.
7. Set the pull force positive as follows:

a. On the menu bar, select **Logger** > **Preferences**.

b. In the **Force positive direction** drop-down list, select **Force - Pull positive**.

c. Tap **OK**.

8. On the main toolbar, tap **Setup**  . The Setup dialog box is displayed.

9. On the **Sensors** tab, tap **Properties**  . The Sensor Properties dialog box is displayed.

10. On the **Set Zero** tab, select **Set the current reading to zero** check box.

11. To return to the Setup dialog box, tap **OK**.

12. On the **Rate** tab, in the **Rate** drop-down list, select **10 samples per second**.

13. On the **Samples** tab, in the **Samples** drop-down list, select **500**.

14. Tap **OK**.

Equipment Setup Summary

Preferences Dialog Box

Force positive direction	Force - Pull positive

Setup Dialog Box

Sensors Tab	
Input 1	**Distance**
Distance Sensor Properties: Set Zero Tab	
Set the current reading zero	**Checked**
Rate Tab	
Rate	**10 samples per second**
Samples Tab	
Samples	**500**

Experimental Procedure

1. Block the funnel's bottom opening with your finger and fill it with water.
2. To begin data collection, on the main toolbar, tap **Run**  .
3. Remove you finger from the funnel and allow it to empty.
4. After the funnel is fully drained, stop collecting data by tapping **Stop**  on the main toolbar. MultiLab displays the collected data on the graph.

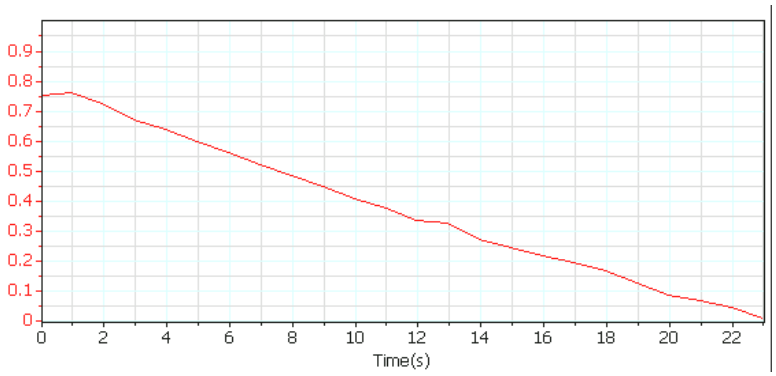

Figure 1-2. . Sample graph

5. Review the graph. If you are satisfied with it, save the data by tapping **Save**  on the main toolbar. Otherwise return to step 1.

Data Table

Quantity	Value
b, the y-intercept	0.756N
Δx	20s
Δy	-0.679N
m, the slope	-0.03395N/s
Calculated linear function	f(x)=-0.033x+0.765

Automatic linear fit	$f(x) = -0.034x + 0.730$

Data Analysis

Now you can analyze the collected data by performing tasks as described in the following sections:

- Trimming Data
- Manually Calculating the Slope and Y-intercept
- Plotting the Manual Line Fit on the Graph
- Applying Automatic Curve Fit to Your Distance versus Time Plot

Trimming Data

After you collected the data, you can select a specific part of the graph that contains the data with which you want to work and cut out irrelevant data. This operation is known as data trimming.

To trim data:

1. On the main toolbar, tap **First cursor**  . The first cursor is displayed.
2. Drag the cursor to the point where the linearly decreasing part of the graph begins.
3. On the main toolbar, tap **Second cursor**  . The second cursor is displayed.
4. Drag the cursor to the point where the linearly decreasing part of the graph ends.

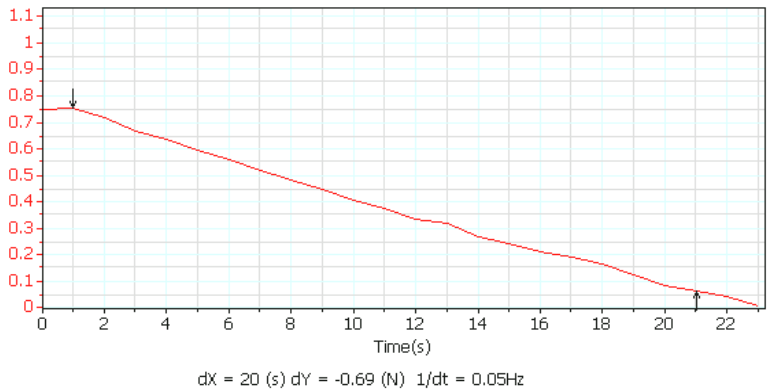

Figure 1-3. Using the cursors to trim unwanted data

5. On the menu bar, select **Tools > Crop**. The time scale is shifted as shown on Figure 1-4.

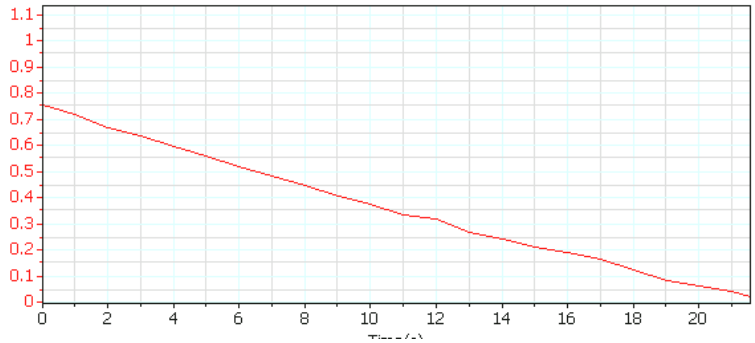

Figure 1-4. Trimmed data

6. To preserve the current graph, on the graph toolbar, tap **Add graph to project**  .

7. On the main toolbar, tap **Save**  .

Note: Notice that a new graph icon has been added to the **Data Map** under the graph category.

Manually Calculating the Slope and Y-intercept

Slope is the change in the y-coordinates divided by the change in the x-coordinates. Coordinate values of a point are displayed in the information bar when you place a cursor onto this point. In this task, you will use the **First Cursor** and **Second Cursor** tools to obtain the coordinate values.

To calculate the slope and y-intercept:

1. On the graph toolbar, tap **First cursor**  . The first cursor is displayed.
2. Drag the cursor to the y-intercept point and record the y-coordinate value in your data table.

Note: The point coordinates appear in the information bar at the bottom of the graph window.

3. On the graph toolbar, tap **Second cursor**  . The second cursor is displayed.
4. Place the cursors at two distinctly separated points as shown on Figure 1-5. Δx and Δy are displayed in the information bar.

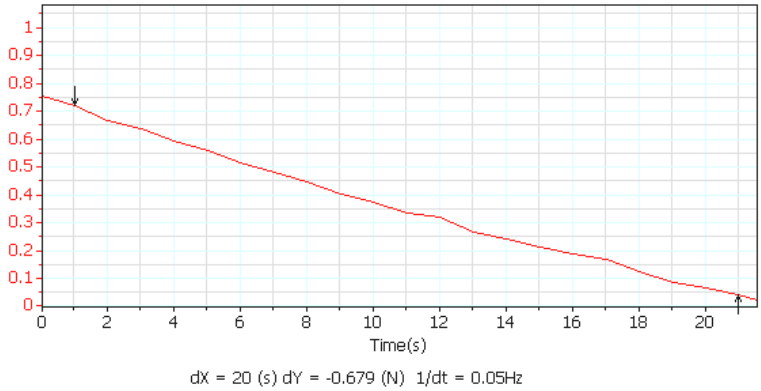

Figure 1-5. Line's slope

5. Record the values Δx and Δy in your data table.
6. Calculate the slope using the equation (1) and record the result in your data table.
7. Drag the cursor to the y-intercept point and record the y-coordinate value in the data table. **The point coordinates are displayed in the information bar.**
8. Record the linear function based on the calculated slope and interception in your data table.

Plotting the Manual Line Fit on the Graph

Based on the results of manual slope and y-intercept calculation, you can now manually plot the line that fits the data contained on the graph.

TIP: Manual calculation uses only one data point to calculate the intercept and two points for the slope.

To plot the manual line fit:

1. On the main toolbar, tap **Functions**  . The Functions dialog box is displayed.
2. In the **Unit** field, enter **N**.
3. In the **Functions** drop-down list, select **Linear**.
4. In the **G1** drop-down list, select **Time.**
5. In the **A** field, enter the calculated slope value from your data table.
6. In the **B** field, enter the y-intercept value from your data table.
7. Select the **Synchronize scale with** check box and select **Cropped data** in the drop-down list.
8. Tap **OK.** The data with linear function is displayed. The fit equation is $f(x) = -0.03395x + 0.756$

TIP: The default function name is its formula. You can rename the function in the **Name** edit box

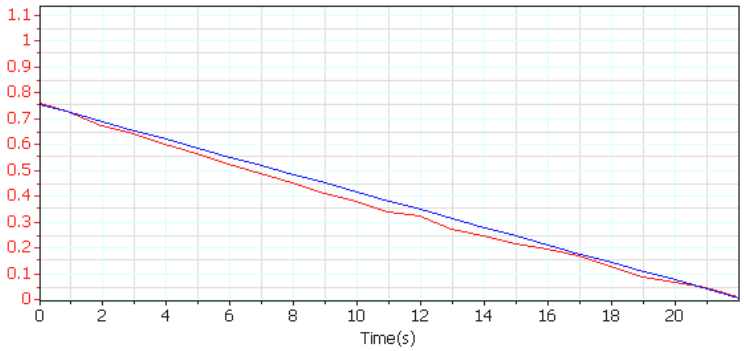

Figure 1-6. Data with linear function

9. To preserve the current graph, on the graph toolbar, tap **Add graph to project**  .
10. Tap **Save**  .

Applying Automatic Curve Fit to Your Distance versus Time Plot

In addition to the manual method of plotting a liner fit, you can use the automatic method.

> **TIP**: Unlike manual fit, the automatic fitting process takes into account all the data points on the graph.

To apply automatic linear fit:

1. In the **Data Map** pane, select **fx linear (cropped data force)**.
2. In the **Data Map** pane, tap **Hide**. Only the cropped data is displayed.
3. On the main toolbar, tap **Linear fit**  . The data with automatic linear fit is displayed. The linear fit equation is displayed in the information bar.

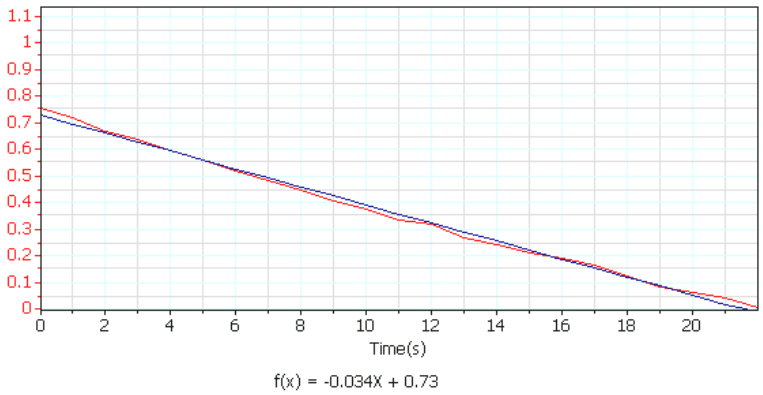

Figure 1-7. Data with automatic linear fit

4. Record the fit equation in your data table.

Questions

Manually calculating the slope and y-intercept

1. Discuss the physical meaning of the slope and the y-intercept.

> The y-intercept indicates the force exerted on the force sensor when data collection begins ($t = 0$). This force is equal to the water weight in the funnel.

The slope indicates the rate of change of the function. In our case it's the change in weight per unit time or the water flow.

2. What are the units of the y-intercept and the slope?

The units of the y-intercept are the units of the Force sensor measurements – N (Newton).

The units of the slope are derived from equation 1:

$$\text{units of } m = \frac{\text{units of } \Delta y}{\text{units of } \Delta x} = \frac{\text{N}}{\text{s}}$$

3. Compare the actual data graph with the linear fit. Is linear function an appropriate model for this activity?

We can see that the linear function is a good approximation to the actual water flow curve.

4. Can the linear model describe a real physical situation?

A linear relation between time and position is an expression of draining with constant flow.

Applying automatic curve fit to your distance vs. time plot

1. Compare the y-intercept and the slope obtained by the automatic fit to the manually calculated values. Why are they not identical?

The slope and intercept of the automatic fit are similar to the calculated values but not identical. The automatic fitting process takes into account all the data points while in my manual calculation I used only one data point to calculate the intercept and two points for the slope.

2. If the initial weight of the water were 10N and the water flow through the opening stays the same, how long it will take for the funnel to completely drain?

Let us substitute $m = -0.03395$ and $b = 10$ in equation (1) to get:

$$f(x) = -0.03395x + 10$$

We now solve it for $f(x) = 0$ to get $x = 294.56\text{s} \approx 3\text{min}$