RCL Resonance

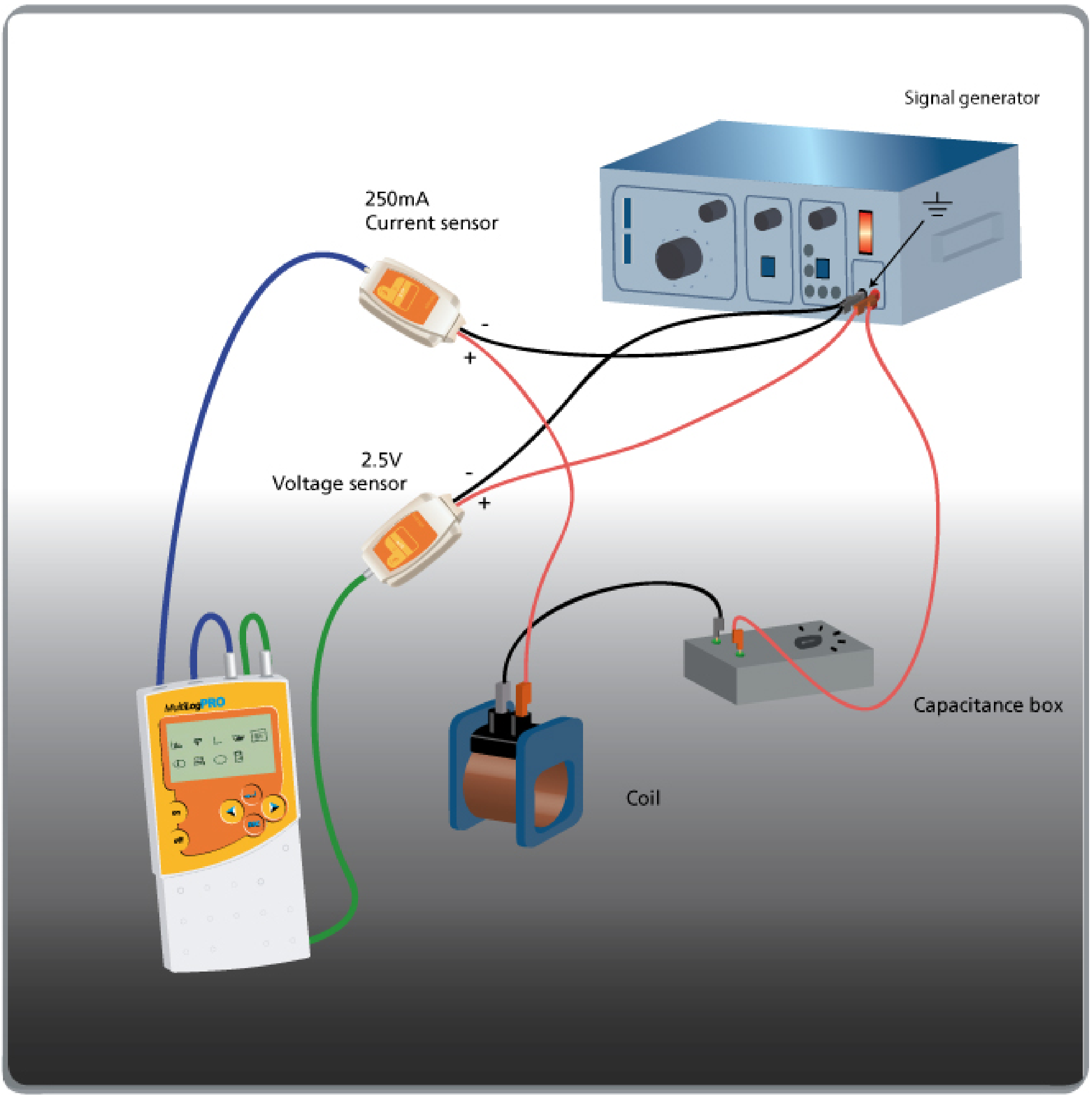


Figure 1

Introduction

When a capacitor, a solenoid and a resistor are serially connected to an AC voltage source the impedance of the circuit is given by the following formula:

$$z = \sqrt{R^2 + (2\pi f L - \frac{1}{2\pi f C})^2} \qquad (1)$$

Where *Z* is the impedance, *R* is the resistance of the circuit, *C* is the capacity, *L* is the inductance of the solenoid and *f* is the frequency.

The impedance is minimal when the frequency is:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad (2)$$

f_0 is the resonance frequency.

At the resonance frequency the current and the voltage are in phase and the power dissipated by the resistor is maximal.

The width of the resonance Δf is defined as the difference between the frequencies f_1 and f_2 at which power dissipation drops to half its maximum value.

The quality factor of the circuit is defined as:

$$Q = \frac{f_0}{\Delta f} \qquad (3)$$

And can be shown to be equal to:

$$Q = \frac{2\pi f_0 L}{R}. \qquad (4)$$

In this experiment these formulas are explored and verified.

Equipment

- Coil. (L~20mH, R as low as possible)
- Box of capacitors
- Box of resistors
- Signal Generator with power output and continuous frequency control
- MultiLogPRO or Nova or TriLink data logger
- Voltage sensor (±2.5 V)
- Current sensor ± 250 mA. (If you don't have the appropriate current sensor you can use a 10Ω resistor and measure the potential difference between its terminals with a second voltage sensor)

Equipment Setup Procedure

1. Connect the data logger to the serial port of the computer.
2. Turn on the data logger.
3. Connect the Voltage sensor to the I/O 1 port of the data logger.
4. Connect the Current sensor to the I/O 2 port of the data logger.

5. Assemble the electric circuit as shown by figures 1 and 2

> ***Note**: The polarity of the sensors is very important when it comes to measuring the relative phase of the current and the voltage.*

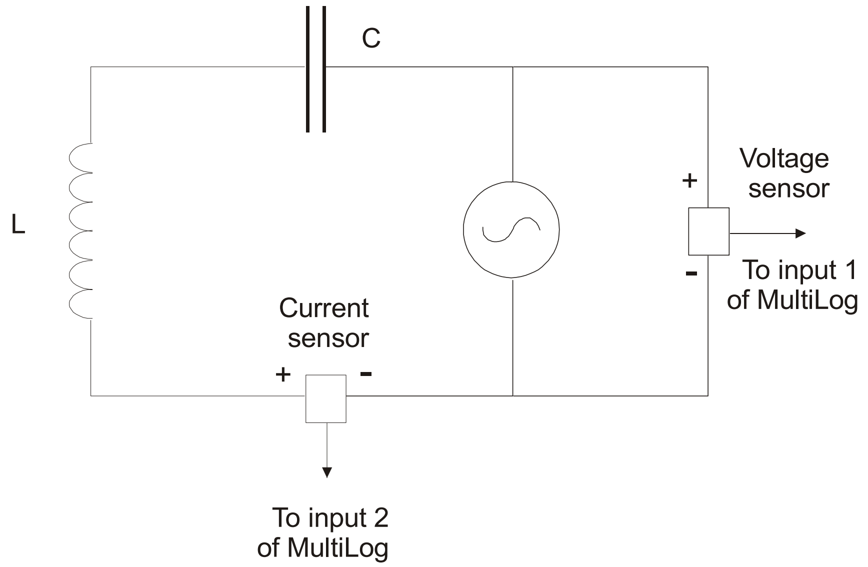


Figure 2

6. Click **Setup Wizard**  on the main toolbar and program the data logger according to the setup specified below

Data Logger Setup

Sensors

Input 1: Voltage ±2.5V (or ±25V)

Input 2: Current ±250mA

(or 10 Ω Resistor + Voltage sensor in parallel)

Rate:

3701 samples per second

Recording time:

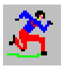
1.35s (5000 samples)

Experimental procedure

1. Turn on the signal generator.
2. Select a capacitor of ~2μF.

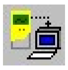
3. Estimate, using equation (2), the value of the resonance frequency (1 kHz depending on the solenoid and the capacitor you selected).

4. Select the signal generator's frequency range that contains this frequency. Set the frequency to the lowest value in the selected range.

5. Click **Run**  on the upper toolbar to begin recording data

6. Sweep the frequency range from the lowest value to the uppermost value. Notice that the duration of data requisition is less than one second and you have to perform a complete scanning of the frequency range in this time.

7. Wait until logging ends. Downloading to the PC will start automatically

Note: If you are using MultiLog v.6 click **Download**  on the main toolbar to download the data.

8. You may want to repeat the experiment several times until you get satisfactory results. The voltage and the current will be displayed in the same window, we show them separately in figures 3 and 4.

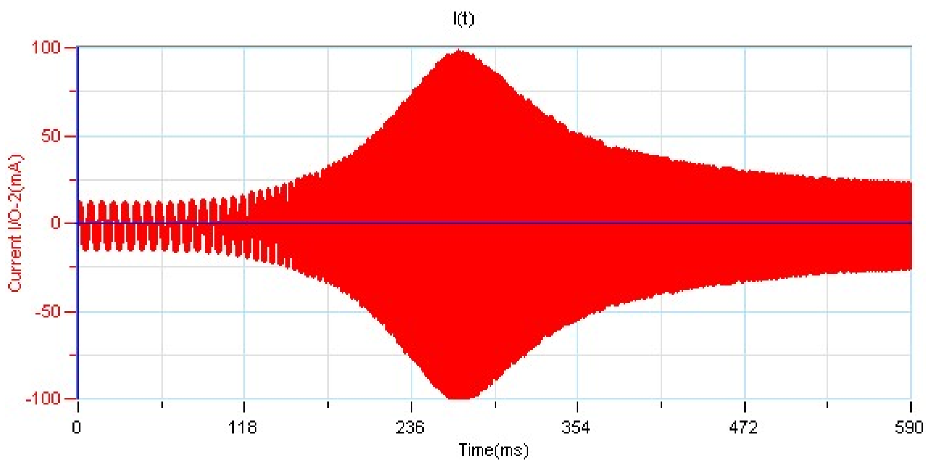


Figure 3

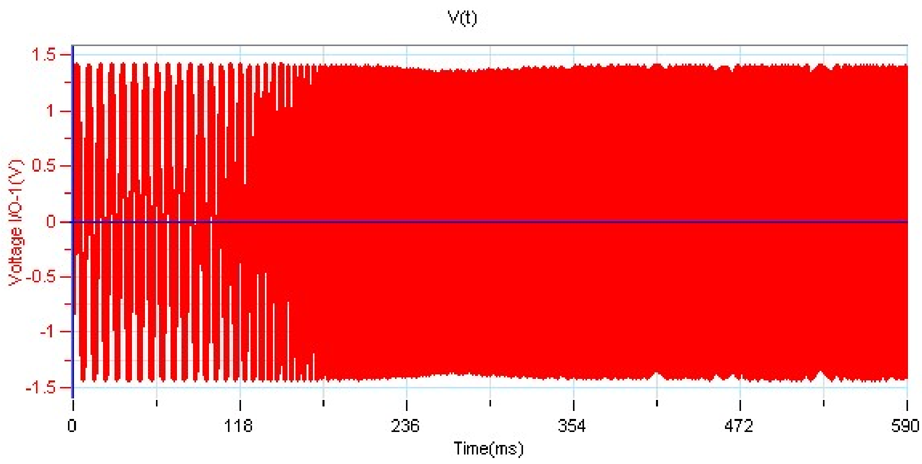


Figure 4

Data Analysis

1. Apply a Fourier transform to the current:

1.a. Click Analysis **Wizard** on the main toolbar, then click the **Functions** tab

1.b. In the **Functions** drop list select **Fourier Transform**

1.c. In the **G1** drop list select current I/O 2, then click **OK**

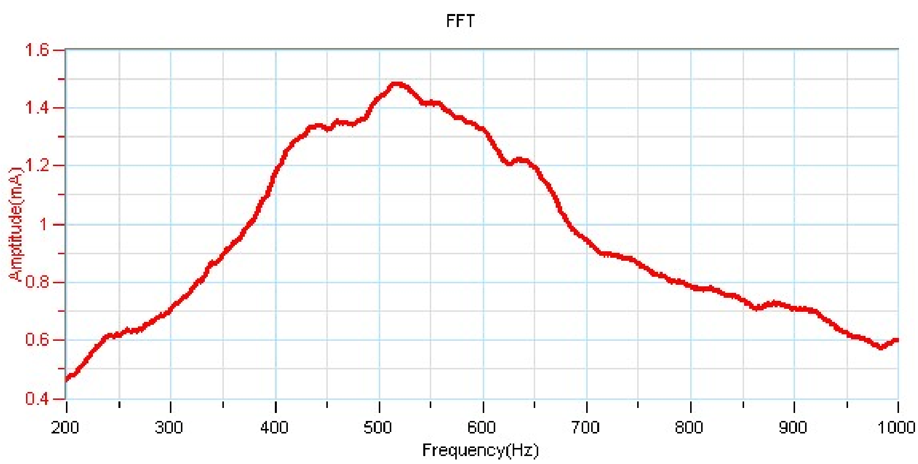


Figure 5

2. Use the cursor to measure at what frequency the graph reaches its peak. This is the resonance frequency. Record this value in

your notebook and compare it to the theoretically expected value.

3. Measure I_0 the maximum value of the transform and record this value in your notebook.

4. Measure the width of the resonance: Calculate the value of $\frac{I_0}{\sqrt{2}}$.

   Use the cursors to measure f_1 and f_2 the frequencies for which the value of the current reaches the value that you have just calculated and record these values in your notebook.

5. Calculate *Q* the quality factor of the circuit:

$$Q = \frac{f_0}{|f_1 - f_2|}$$

6. Use *Q* the quality factor you have calculated and formula (4) from the introduction to calculate the resistance *R* of the circuit.

7. Explore the influence of the resistance on the resonance:

7.a. Add a resistor to the circuit in series with the current sensor.

7.b. Repeat the experiment.

7.c. Repeat stages 1 to 6 of the data analysis and compare the new values with the values you recorded before.

7.d. Use the new value of *Q* to estimate the new resistance of the circuit and compare this value to the value of the resistance that you added to the circuit.

7.e. Remove the resistor you added to the circuit.

8. Explore the influence of the capacity on the resonance frequency:

8.a. Change the value of the capacitor.

8.b. Repeat the experiment.

8.c. Repeat stages 1 and 2 of the data analysis.

8.d. Repeat this procedure with five different values of capacity.

8.e. Use the capture tool to create a graph of f^2 vs. $\frac{1}{C}$. The graph should be linear and the slope should equal:

$$\text{slope} = \frac{1}{4\pi^2 L}$$

8.f. Use the value of the slope to calculate the inductance of the circuit.

Further Suggestions

1. You can repeat the experiment with different solenoids.
2. In the original graph of the voltage and the current vs. time you can zoom in on different parts of the graph and observe the relative phase of the current and the voltage at different frequencies. At the resonance frequency the current and the voltage are in phase.